Graph algorithms and models – LTCC 2019
Project Topic 4 – Random graphs

18th March 2019

Introduction

A central question in network science is whether the peculiar structural properties of the real-world networks we observe are peculiar at all, or if instead they would arise naturally is a class of random graphs. Hence, finding mathematical models to explain the emergence of such properties, such as small characteristic path length, high clustering, heavy-tailed degree distibutions, is still a quite active line of reasearch. If you like to ask yourself "why" things work as they work, then this topic is for you. Choose one or two tasks from the list below for your project.

Proposed tasks

1. Prove that the degree distribution of an Erdos-Renyi graph is a Binomial.

2. Prove that the linear preferential attachment model generates graphs with a power-law degree distribution $P(k) \sim k^{-3}$ for $N \to \infty$. *(Hint: you can use either the master equation for the expected number of nodes with a certain degree or the expression for the expected degree of a node over time.)*

3. Write an original program (using a programming language of your choice) to generate Watts-Strogatz small-world networks. Use your program to show how the small-world effect emerges as the edge rewiring probability increases.

4. Write an original program (using a programming language of your choice) to grow graphs according to the generalised preferential attachment rule:

$$\pi_{n \to i} \propto k_i^{\alpha} \quad (1)$$

   In this model, the probability for the new node n to create a link to the existing node i is proportional to the degree k_i of node i to the power α. Use your program to generate a sequence of random graphs with N nodes and $\langle k \rangle = 4$, for different values of α in $[0, 2]$. How does the degree distribution of the resulting graph change as a function of α? Find a smart way to present your results.

5. Use an existing program (or write your own) to create a random graph with $N = 2^{2n}$ nodes (for $n \geq 5$), $K = 6N$ edges, and 4 communities of size $C = N/4$. In this model, all the nodes have the same expected degree $z = 12$, and each node has an expected number $z_{out} < 12$ of edges towards nodes in other communities, while its remaining $z_{in} = z - z_{out}$ edges are to nodes inside the same community. Use your program to generate a sequence of graphs with different values of $z_{out} \in [0, 12]$, and report in a plot the value of the modularity function over z_{out}. *(Hint: this model is also know as the Newman-Girvan benchmark).*

6. Use an existing program (or write your own) to generate Erdos-Renyi graphs having the same number of nodes and edges as the following graphs:

   - The Internet Autonomous Systems network
     `http://www.complex-networks.net/datasets/Internet_AS.zip`

- The Astrophysics coauthorship network
  http://www.complex-networks.net/datasets/coauthorship_astro-ph.zip
- The US air transport network
  http://www.complex-networks.net/datasets/US_airports.zip

Compare the structural properties of the original networks with those of the equivalent Erdos-Renyi random graphs. Present and discuss your results.